6. ATOMIC, NUCLEAR & QUANTUM PHYSICS

ATOMIC AND QUANTUM PHYSICS (SECTIONS 6.1 - 6.7)

6.1. The atomic model

During the 1800s the idea that all materia consists of atoms gradually gained support. Its fundamental particles are, as we today describe it, the electrically positive proton ($p^+$) and neutral neutron (n) which both are ca 2000 times heavier than the negative electron ($e^-$). The electrons occupy certain levels or "shells" outside the nucleus with the protons and neutrons.

Evidence for the idea that the positive charges in the atom are not spread evenly in it (the Thomson model) is given by the Geiger-Marsden (or Rutherford) experiment in which positive particles (alpha particles from decaying atoms = helium nuclei, see later) are aimed at a thin gold foil. The result is that most of them pass through the foil without changing direction, while some bounce back in almost the same direction as they came from. This can be compared to firing a machine gun at a fence made of solid iron bars with wide openings between them, as opposed to firing it against a wooden fence with no openings.
The radius of an atomic nucleus can be found by letting positive particles with a known kinetic energy be scattered (bounce from) the nuclei. The "closest approach" is found by equating their kinetic energy with the electrical potential difference they need to overcome:

- Kinetic energy: \( E_k \)
- Potential far from nucleus: \( V_1 = -\frac{kq_{\text{nucleus}}}{r_1} \), where \( r_1 \gg 0 \) and \( V_1 = 0 \).
- Potential at closest approach distance from center of nucleus = \( V_2 = -\frac{kq_{\text{nucleus}}}{r_2} \), where \( r_2 = r_{\text{nucleus}} \).
- Potential difference = \( V_1 - V_2 = \frac{kq_{\text{nucleus}}}{r_{\text{nucleus}}} \)

Work done against repulsive electrical field of nucleus = \( E_k = q_{\text{particle}}\Delta V = k\frac{q_{\text{particle}} q_{\text{nucleus}}}{r_{\text{nucleus}}} \)

\[ => r_{\text{nucleus}} = k\frac{q_{\text{particle}} q_{\text{nucleus}}}{E_k} \]
It can be found that the radius of the nucleus is far smaller (ca $10^{-14}$ m) than the size of the whole atom ($10^{-10}$ m) which therefore can be compared to a solar system with a central body (sun/nucleus), smaller bodies around it (planets/electrons) and nothing in between. Other experiments (Thomson and Millikan) showed that the mass of the electron is much smaller (ca 1/2000 of) than that of a hydrogen nucleus. The discovery that the nucleus besides positive charges (protons) also contains neutrons will be returned to later (mass spectrometer section).

6.2. Spectra and energy levels in atoms

Emission spectra

If a small amount of a pure gas is placed in an airtight tube and this tube is heated e.g. by letting an electric current pass through it, it will emit some light. If this light is led through collimators and a diffracting prism the wavelengths of light present can be studied. Unlike white light (or sunlight) only certain wavelengths will be present.

Absorption spectra

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If white light with all wavelengths present passes through the gas in the tube, most of it will pass through but some wavelengths will be missing - the same ones as were emitted from the heated gas!

An interpretation of this is the following:

- the electrons in atoms can be in certain "shells" numbered \( n = 1, 2, 3, \ldots \) or indicated with letters K, L, M, N, ... (K = 1, L = 2 etc)
- when an electron falls from a higher to a lower level, the difference in energy between the shells can be sent out as a photon
- photon = "particle" of light or other electromagnetic radiation. The energy of the photon is: \( E = hf \)
- in an emission spectrum: since there are only certain possibilities to fall in an atom (shell 2 \( \rightarrow \) 1, 3 \( \rightarrow \) 1, 4 \( \rightarrow \) 1, ..., 3 \( \rightarrow \) 2, 4 \( \rightarrow \) 2, ...) only certain frequencies are emitted, and these are typical for a certain element)
- the "falls" can be called \( K\alpha, K\beta, \ldots, L\alpha, L\beta, \ldots \) where the letter says to which shell an electron falls, and \( \alpha \) that it falls from the next higher, \( \beta \) from one two shells higher etc. For example \( L\beta \) means it falls to shell 2 from shell 4.
- the higher the energy difference between the shells (can be illustrated as an energy level diagram) the higher the energy of the photon and the higher the frequency or the lower the wavelength of the emitted light.
- absorption spectrum: if white light with all possible wavelengths present passes through a vapour of an element, the photons will be absorbed, = disappear and turn to energy lifting an electron to a higher shell (from which it probably will fall back down later). Then the frequencies which would have been emitted in falling are now missing from the spectrum.
It can be shown [Planck's studies of blackbody or cavity radiation (not in the IB), supported by the photoelectric effect (later)] that the energy of a photon, a "particle" of light is

\[ E = hf \quad [DB \ p. \ 8] \]

where \( f \) = the frequency of the light or other EM waves, and \( h = \text{Planck's constant} = 6.63 \times 10^{-34} \text{ Js} \)

Since the speed of light = \( c = f \lambda \) = constant in vacuum = 300 000 000 ms\(^{-1}\) we also get \( f = c/\lambda \) and

\[ E = \frac{hc}{\lambda} \]

**The empirical Rydberg formula**

For **hydrogen** atoms it was found experimentally that the wavelength \( \lambda \) of the emitted light follows

\[ \frac{1}{\lambda} = R_H (1/n^2 - 1/m^2) \quad [DB \ p \ 11] \]

where \( n = \) the number of the shell the electron falls to, \( m = \) number it falls from and \( R_H = \) the Rydberg constant = \( 1.096 \times 10^7 \text{ m}^{-1} \) (not the gas constant \( R \), not in data booklet, given when needed).

**Example:** What frequency is emitted by hydrogen in a \( L_\gamma \) emission?

Solution: \( L_\gamma \) means the electron falls to shell 2 from shell 5 so

\[ \frac{1}{\lambda} = R(1/2^2 - 1/5^2) = R(1/4 - 1/25) = 0.21R \Rightarrow \lambda = 1/0.21R = 1/(0.21 \times 1.096 \times 10^7 \text{ m}^{-1}) = 4.3448 \times 10^{-10} \text{ m} \]

and since \( c = f \lambda \Rightarrow f = c/\lambda \) we get

\[ f = 300 000 000 \text{ ms}^{-1} / 4.3448 \times 10^{-10} \text{ m} = 6.9 \times 10^{17} \text{ Hz} \]

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6.3. The Bohr model of the atom

Bohr's assumption

Earlier the Rutherford experiment (also called Geiger-Marsden experiment) had shown that most mass in the atom is in the positive nucleus. But electrons orbiting the nucleus should be sending out radiation (since they are accelerated, according to theory not required in the IB programme) and then lose energy and collapse into the nucleus. This does not happen, and Bohr assumed the following:

- only certain orbits with certain radius values are allowed and in these the electron is stable
- the smallest value, \( r_1 \) for shell \( n = 1 \) (the K-shell) is the "ground state". If an electron is higher up and there would be room for it in a lower shell, it is in an "excited" state.
- the allowed radius values are such that the angular or rotational momentum \( L \) is

\[
mvr_n = \frac{nh}{2\pi} = nh' \quad \text{[not in DB]}
\]

where \( m \) = mass of electron, \( v \) = speed of electron, \( n \) = shell number, \( h \) = Planck constant.

[It can then be shown that the same formula for the emitted wavelength as Rydberg found experimentally is true: First, for the possible radii of the orbits of electrons in an H-atom where the nucleus and the electron have the same charge \( q (=e) \) though with opposite signs:

- the centripetal force on an electron = the Coulomb force so \( mv^2/r = kq^2/r^2 = kq^2/r^2 \) where \( k \) = the Coulomb constant
- solving for \( r \) gives \( r = \frac{kq^2}{mv^2} \) and using \( mv = nh' \) with \( h' = h/2\pi \) which gives \( v = \frac{nh'}{mr} \) we get:
  - \( r = \frac{kq^2}{(mnh'/mr)^2} \) which becomes \( r = \frac{kq^2m^2}{n^2h'^2} \), solving for \( r \) gives:
  - \( r = \frac{h^2n^2}{mkq^2} \) or for shell \( n : r_n = \text{constant} \ A * n^2 \)

Then to find the energy levels we note that an electron, like a satellite in orbit around a planet, has both a kinetic and a potential energy which is negative:

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• \( E = E_k + E_p = \frac{1}{2}mv_2^2 + (-kqq/r) = \frac{1}{2}mv_2^2 - kq^2/r \) into which we put \( v = nh'/mr \) so

\[ E = \frac{1}{2}m\left(\frac{nh'}{mr}\right)^2 - kq^2/r \]

• here we have \( \text{const.B} = \frac{mkq^4}{2h'^2} \)

For \( n = 1 \) it turns out that \( E = -13.6 \) electronvolts, the energy needed to ionise a hydrogen atom with its electron originally in the lowest shell. To get towards the Rydberg formula we look at an electron falling from shell \( m \) to shell \( n \) (or being raised from \( n \) to \( m \)), where \( m > n \):

• the change in energy = \( E_n - E_m = \text{constB}/n^2 - (\text{constB}/m^2) \) which is

\[ \text{constB} \left( \frac{1}{m^2} - \frac{1}{n^2} \right) \]

When the atom loses this energy, a photon with the energy \( E = \hbar f \) is emitted. The change in energy of the atom is negative and that of the photon is positive (so totally the energy is conserved). We can equate them after adjusting the sign:

• \( -\hbar f = \text{constB}^* \left( \frac{1}{m^2} - \frac{1}{n^2} \right) \) or \( \hbar f = \text{constB}^* \left( \frac{1}{n^2} - \frac{1}{m^2} \right) \). Combining with \( c = f\lambda \)

\[ \frac{hc}{\lambda} = \text{const.B}^* \left( \frac{1}{n^2} - \frac{1}{m^2} \right) \] or dividing with \( hc \) then

\[ \frac{1}{\lambda} = (\text{const.B}/hc)^* \left( \frac{1}{n^2} - \frac{1}{m^2} \right) \]

Finally we note that:

• \( \left( \text{const.B}/hc \right) = \frac{mkq^4}{2h'^2}/hc \). Inserting the values gives the same value for this expression as that of the experimentally found Rydberg constant \( R_H \).

[Or to be more precise: the value so reached is close enough to the experimental one to show that we are on the right track with this theory, but leaves a discrepancy not accounted for by conventional experimental uncertainties. This lead to the development of the radically different way of describing the physic world known as quantum mechanics, using which the discrepancy disappears.]

The limitations of the Bohr model are:

- why do the electrons stay in their orbits - they are centripetally accelerated and should emit energy, and lose potential energy and collapse into the nucleus?
- why should the assumption about the rotational momentum be made?

6.4. De Broglie and wave-particle dualism

But why would \( mv_n = nh/2\pi \)? The answer is in looking at all material (in this case electrons) as being a particle and and a wave at the same time (earlier we have defined light as waves and "photon" particles at the same time - more support for this later).
The double slit experiments: Young and Davission-Germer

A reason for looking at electron as waves is that if they are sent towards two or more very narrow slits they do not just end up right behind the slits but bend to the same directions as laser light would in a Young's experiment (remember the \( d \sin \theta = n \lambda \) formula!).

De Broglie assumed that the electron (and all other particles) would at the same time be waves with the wavelength

\[
\lambda = \frac{h}{p} \quad \text{[DB p. 8]}
\]

where \( h = \text{Planck's constant} \) and \( p = \text{the momentum} = mv \)

[The reason for this is the theory of relativity, which says that for the total energy \( E \) of something \( E^2 = m_0^2 c^4 + p^2 c^2 \) which for massless photons becomes \( E^2 = p^2 c^2 \) or \( E = pc \) which with the photon energy \( E = hf \) gives \( pc = hc/\lambda \) so \( p = h/\lambda \) or \( \lambda = h/p \). Assuming that the same is true for electrons in the double slit experiment gives a \( \lambda \) which fits the usual \( d \sin \theta = n \lambda \) formula.]

Electrons as standing (stationary) waves around the nucleus

A wave can exist for a longer time in a certain place if it is a standing wave, like that on a guitar string. In an atom where no air resistance or friction damps it, it can remain standing "forever". An electron wave must be bent around the nucleus:
To fit in the electron wave must follow the condition \( n\lambda = 2\pi r \Rightarrow nh/p = 2\pi r \Rightarrow nh/mv = 2\pi r \Rightarrow mvr = nh/2\pi \) which was Bohr's assumption (originally made only because it happened to lead to a formula which fits Rydberg's experiments).

6.5. The Schrödinger model of the atom

*From de Broglie waves to the wavefunction \( \Psi \) (psi)*

Above we have described particles, like an electron, with waves as in the de Broglie wavelength \( \lambda = h/p \). In this we use one concept relevant to waves (wavelength) to give information about one quantity relevant to the particle (momentum). In modern quantum physics, this has been extended so that a "wave function" \( \Psi \) is used to describe the particle. The equation for an ordinary travelling wave can be written (with \( A = \) amplitude) for example as \( y(t) = A\sin(2\pi ft + \) phase shift) or \( y(x) = A\sin(2\pi x/\lambda + \) phase shift). In the wave function sine and cosine functions are also used, but the variable and also the function can have imaginary or complex values (of the form \( a + bi \), where \( a \) and \( b \) are real numbers and \( i \) the imaginary number unit, where by definition \( i^2 = -1 \)). The wave function can be very complicated but for a particle moving in one dimension along the x-axis it can be something like

\[
\Psi(x) = \cos(kx) + i\sin(kx) = e^{ikx}
\]

where \( k = \) the wave number \( = 1/\lambda \). The complex numbers mean that it is difficult to describe where a particle is directly with the wave function, since a probability should be a positive real number. What is used instead is the square of the absolute value of it. If we focus on one dimension, the probability of finding the particle in the interval \( \Delta x \) of the x-axis will be:

probability of \( x \) being in \( \Delta x \) at the time \( t = |\Psi(x,t)|^2 \Delta x \)

which was under the assumption that \( \Psi \) has a constant value in this interval, which rarely is true. One will then use an infinitely short interval \( dx \) where the probability is \( |\Psi(x,t)|^2 dx \). (The \( \Psi(x,t) \) function is called the probability density function).

Since the particle with a probability of 100\% = 1 is somewhere on the x-axis (in the one-dimensional case) summing up all the probabilities from all small intervals from negative infinity to positive infinity will be \( = 1 \). If it isn't, the wave function can be renormalised by introducing a suitable constant into it. Then we will have (for a sum of infinitely many infinitely small intervals)

\[
\int_{-\infty}^{+\infty} |\Psi(x,t)|^2 \, dx = 1
\]
Quantities as operators

A function is something that transforms one number into another, for example the function \( y = x^2 \) turns 1 into 1, 2 into 4, 3 into 9, 4 into 16 etc. An operator is something which transforms one function into another function. The integral and derivative are operators; for example the operator derivative transforms \( y = x^2 \) into \( y' = 2x \). But there can be many other operators (although many of them do contain some kind of derivatives) and most physical quantities can be represented by operators operating on the wave function. For example the quantity momentum will (in one dimension, the x dimension) be:

\[
p = ih'd/dx
\]

(that is, the x-derivative multiplied by \( ih/2\pi \) using \( h' = h/2\pi \). In most books \( h' \) would be symbolised by an \( h \) with a bar through its vertical part, an called "h bar").

The Hamilton (total energy) function and operator

The most important operator is the one which describes the total energy of a particle and therefore can be used to give as much information as possible about the particle and its future. Classically, the Hamilton function is

\[
H = E_{\text{tot}} = E_k + E_p
\]

which also can be written (since \( p^2/2m = m^2v^2/2m = ½mv^2 = E_k \))

\[
H = p^2/2m + V \text{ or better: } H = p^2/2m + E_p
\]

where the "\( V \)" is often called potential (and may change both with place and time) although this is not very good since it cannot be a potential (unit Jkg\(^{-1}\)) and still added to an energy (unit J). In this section \( V \) is then used as another symbol for \( E_p \), unlike in the Mechanics where it was \( V = E_p/m \) or the Electricity sections where \( V = E_p/q \). We can note that in the \( p^2/2m \)-term \( m \) is the inertial mass while the \( V \) stands for any potential energy, related to gravitational force, electrical force or other (e.g. strong interaction or a combination of several forces). Compare this to the difference between inertial and gravitational mass.

As an operator \( p^2/2m \) then becomes \((-h'^2/2m)d^2/dx^2 \) and the whole Hamilton operator then

\[
H = (-h'^2/2m)d^2/dx^2 + E_p(x,t)
\]

The Schrödinger equation

A differential equation is one where functions and their derivatives (and possibly other things, like various constants) are included. The solutions to differential equations are functions which satisfy them.

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All that can be known about a particle is expressed in the Schrödinger equation, which is a differential equation:

\[ \text{i} \hbar (d \Psi /dt) = H \Psi \quad \text{or} \]

\[ \text{i} \hbar (d \Psi /dt) = (-\hbar^2/2m)d^2\Psi/dx^2 + E_p(x,t) \quad \text{[not in DB]} \]

The Schrödinger equation in one dimension can use either only a function of the position of the particle \( \Psi(x) \) - the time-independent Schrödinger function - or of both the position and the time \( \Psi(x,t) \) - the time-dependent function, which describes all that can be known about the particle now and in the future. In 3 dimensions we have \( \Psi(x,y,z) \) and \( \Psi(x,y,x,t) \).

**The collapsing wave function**

When a measurement of a physical quantity is done (for example momentum) then the operator \( O \) associated with this quantity will produce a function \( v(x,t) \) for value of the quantity where so that

\[ O \Psi(x,t) = v(x,t) \Psi(x,t) \]

This is a differential equation and the solution is a function which gives the value of the quantity. If the measurement is never done in a certain chosen way, the quantity does not have a well-defined value. Since the wave function can contain terms that represent interference - constructive of destructive - the "particle" can behave in a wave-like way (for example a single electron passing through both slits in a double-slit experiment and interfering with itself to produce a pattern similar to that in a Young's experiment for light). Measuring it and thereby letting it "get" a more defined value (e.g. putting an electron detector where the electrons hit a screen) is called a "collapsing" of the wave function. Finding out what can be found out about the value of a physical quantity would then involve these steps:

- finding the \( E_p \)-function relevant to the situation, and with that the relevant Hamilton operator
- solving the Schrödinger equation (a diff. equation); result: the relevant \( \Psi \)-function
- solving the diff. eq. \( O \Psi = v \Psi \) for the operator for the physical quantity to find the function \( v(x,t) \)
- inserting the relevant \( x \)- (and \( t \)-, if relevant) values in \( v(x,t) \) to find the value (or calculating the value of \( \int |\Psi(x,t)|^2 \, dx \) integrating over the relevant interval to find the probability that the particle is there, that is in the interval \( dx \)).

In 3 dimensions we have \( x,y,z \) or some other 3-dimensional coordinates instead of just \( x \).

**The philosophical question : whether we know or what we know?**
Before the wave function $\Psi$ has been used to find a probability or other value, this value is not well defined and some would say that it does not have a value until we decide to measure it in a certain way (where the way we decide to measure it will affect what value it gets). One interpretation of this is that our conscience "produces" the measurement result. It must, however, be noted that this does not affect the Schrödinger equation itself - it has the properties it has regardless of our decisions and even existence. One can therefore say that quantum physics has not changed the answer to the question of whether we can know things about the world which are independent of our choices, it has changed the answer to another question: What, independent of our choices, can we know? The answer is no longer the values of physical quantities but the Schrödinger equation.

**The Heisenberg uncertainty relations**

Measuring two quantities at the same time cannot always be done with as high accuracy as wanted. In quantum physics this is not only because of practical difficulties in the measurement but it can be mathematically shown that certain pairs of quantities cannot be known precisely at the same time. (Quantities => operators and other relevant concepts can also be represented by matrices, mathematical tools which in some ways can be used in a way similar to numbers, but have certain strange properties. The matrices A and B can be multiplied with each other, but unlike for numbers the order of multiplication makes a difference: AB is generally not the same as BA and AB - BA is not zero).

There are many such pairs, but the ones most often mentioned are:
- position and momentum
- energy and time

\[ \Delta x \Delta p \geq \frac{\hbar}{2\pi}, \quad \Delta E \Delta t \geq \frac{\hbar}{2\pi} \] [DB p. 11]

From the latter we get $\Delta E \Delta t \geq \hbar / 2\pi = h' \Rightarrow \Delta t \geq \Delta E / h'$. It turns out that if no measurement-technical difficulties are in the way the uncertainty is close to its lowest limit, so

\[ \Delta t = h'/\Delta E = h'/ (\Delta mc^2) \]

**The virtual ("Cinderella") particles**

The impossibility of knowing the energy of something is relevant also for the law of conservation of energy - that you cannot get energy (or mass as in $E = mc^2$) from nothing. This will have a relevance for the virtual field particles which can "pop into existence" from nothing, pure vacuum, for a certain time which depends on their mass. If they are massless they can do so forever and then the force they are related to can act over infinitely long distances (they cannot move faster than light). If they have a mass, $\Delta t$ is limited and the virtual field particle must disappear (like Cinderella before midnight) and therefore the related force can only act over a limited, usually very short distance. All the cheated "energy from nothing" must disappear so the virtual particle cannot be detected.
since that would mean some interaction with real particles which might get some of that energy in this interaction.

**The H-atom and spherical coordinates => electron shells and subshells**

For the electron in hydrogen atoms (and ions with only one electron left) the Schrödinger equation can be solved algebraically; for other cases, only numerical approximations with computers are possible. We will have $\Psi(x,y,z)$ replaced by another wave function of **spherical coordinates** $\Psi(r,\theta,\phi)$ where

- $r =$ the distance from (the center of) the nucleus
- $\theta,\phi =$ angular coordinates

The angular coordinates can be compared to longitude and latitude coordinates on the surface of earth (for all points on the surface $r =$ the radius of earth; the origin of the coordinate system would be in the center of the earth).

It turns out that the $\Psi$-function can be separated as $\Psi(r,\theta,\phi) = f_1(r)f_2(\theta)f_3(\phi)$ and the three functions solved separately. Each gives solutions (involving sines or cosines, which are functions that can produce a set of discrete solutions, e.g. $\sin x = 0$ gives $x = n*2\pi, n = 0,1,2,...$) for certain whole numbers of a variable ($1, 2, 3, ...$) like the **resonant frequencies of a standing wave** in the Waves section, or the de Broglie description of the electron as a standing wave around the nucleus.

These discrete values (values that do not vary continuously but only can have certain values) can be used to define several different **quantum numbers** for the possible states of the electron. The most known are the ones given by the solutions involving $f_1(r)$ which give the main or radial quantum number $n = 1, 2, 3, ... =$ "the number of the electron shell". Other quantum numbers (there is also a fourth caused by the possibility for the electron to spin either the same or the opposite way as the nucleus) produce subshells (1s, 2s, 2p, etc.).

In this context we may also mention the **Pauli principle**: no electron can have all four quantum numbers identical to another electron in the same atom (since these numbers contain all information we have about the electron, it can only be itself!) which leads to the limitations in how many electrons can be found in the electron shells of an atom.

### 6.6. The photoelectric effect

**Additional support for $E = hf$**

Planck had suggested that photons could be described not only as waves but as wave packages or particles with the energy $E = hf$. Einsteins explanation of the experimental results for the "photoelectric effect" support this.
Thermionic emission and basic photoelectric effect

If we have two metal plates (electrodes) in a vacuum tube, and connect a voltage to the plates, then some electrons will flow from the negative cathode to the positive anode. The reason is that at a temperature above 0 K all electrons have some kinetic energy; some of them enough to break free from the metal and then be accelerated by the electric field. If a plate is hit by light, then the electrons get more kinetic energy, and the current increases.

Reversing the voltage

To investigate the kinetic energy that the electrons get we can reverse the voltage (⇒ we have a "stopping voltage") or potential difference so that it works against the electrons which then can get from one electrode to the other only because of their kinetic energy. Some of this kinetic energy is needed to break the electron free from the metal, the rest is lost to work against the stopping voltage. We also let light hit only one of the plates so that we do not get the same phenomenon working both ways, which would cancel out any current between the plates. By measuring the stopping voltage needed to get the current to zero, we can measure the maximum kinetic energy any electron gets from the light. This, presumably is the energy the light itself has.
We then have that \( qV_{\text{stop}} = eV_{\text{stop}} = eV_s = \frac{1}{2}mv_{\text{max}}^2 = E_{k,\text{max}} \) (= \( KE_{\text{max}} \) in some books) and can study the phenomenon either:

A. as a function of the frequency of the light (different colours)
B. as a function of the intensity of the light (bright or dim light)

**Results of A:**
- The graph is a straight line which hits the \( f \)-axis at some minimum \( f_0 \)
- the slope of the line is \( h = \text{Planck's constant} \)
- the intercept with the \( E \)-axis is always the same, called \( W_0 \), for the same metal in the plate hit by the light

\[
E_{k,\text{max}} = hf + W_0
\]

**Results of B:**
- Increasing the intensity of the light does not affect either \( h, f_0 \) or \( W_0 \)

Interpretation: the energy of the light depends only on \( h \) and \( f \), not on its brightness which it should if it was only a wave. (For a wave the energy transported through a certain area depends on the amplitude, that is how bright the light is. Here we will then have to reinterpret brightness of light as number of photons moving per time).

The graph gives the equation \( E_{k,\text{max}} = hf + W_0 \) if \( W_0 \) is negative or \( E_{k,\text{max}} = hf - W_0 \) if we keep the value positive. (Compare this to a graph of \( y = 3x - 2 \) which has the slope 3 and intersects the \( y \)-axis at -2). Moving terms we get \( hf = W_0 + E_{k,\text{max}} \) or if we as the IB:s data booklet use \( \varphi \) as a symbol for \( W_0 \):

\[
hf = \varphi + E_{k,\text{max}} \quad \text{[DB p. 8]}
\]
or since \( eV_s = E_{k,max} \) we get \( hf = \varphi + eV_s \) and if by \( f_0 \) we mean the frequency value at which the graph intersects the horizontal f-axis, that is where \( E_{k,max} = 0 \), then inserting this in \( hf = \varphi + E_{k,max} \) gives \( hf_0 = \varphi \) so the same formula can be written:

\[
hf = hf_0 + eV_s \quad \text{[DB p.8]}
\]

where \( W_0 \) or \( \varphi = \text{the "work function"} \) = the energy or work needed to free an electron from the metal (different for different metals).

### 6.7. X-rays

**X-ray production and spectral features**

When electrons are accelerated from a negative cathode (which may be heated to increase the number electrons which break away from the cathode) towards a positive anode by a high voltage \( V \) (several kilovolts) they will hit the target atoms and either:

- **A:** release all or part of its kinetic energy as a radiated photon during the deceleration ("Bremsstrahlung"), which is based on the notion that accelerating charges means they lose energy (same as the problem with the Bohr model, where the centripetal acceleration should make them lose energy) or
- **B:** strike out an electron from an inner shell even if there are several electron shells in the atom. When an electron from a higher shell falls down to replace it, photons with certain high energy values are released.

The highest energy \( E = hf \) with the highest \( f \) and the lowest \( \lambda \) since \( E = hc/\lambda \) (so that energy and wavelength are inversely proportional to each other - **minimum wavelength corresponds to maximum energy**) is obtained when all the work by the accelerating voltage = \( qV = eV \) becomes kinetic energy \( \frac{1}{2}mv^2 \) and then photon energy \( hc/\lambda = eV \Rightarrow \lambda = hc/eV \), the shortest emitted \( \lambda \):

\[
\lambda_{\text{min}} = \frac{hc}{eV}
\]
The important features of the X-ray spectrum are then:

1. the minimum of cutoff wavelength, which can be decreased by increasing the accelerating voltage $V$
2. the continuous curve (from A) of X-ray intensity for electrons which give only part of their kinetic energy to radiation of photons (the rest to absorptions related to energy levels which produce other radiation than X-rays)
3. the characteristic peaks from (B) the strikeout electrons replaced by others falling down (the higher the difference between the energy levels of the involved shells, the higher the photon energy => the lower the $\lambda$, so a $K_\alpha$ peak would be to the right of a $K_\beta$-peak since the latter gives more energy)

*X-ray diffraction: Bragg's formula*

Waves with short wavelengths can be diffracted by the atoms in a crystal lattice (Sw. kristallgitter, Fi. kristallihila) as in the graph below.
Note that the angle used is the **grazing angle** = the angle between incoming ray and surface, not incoming ray and normal to the surface as usual. For this the **Bragg formula**: \(2d \sin \theta = n\lambda\) is valid and \(\lambda = h/p\) makes it work also for electrons.

NUCLEAR AND PARTICLE PHYSICS (SECTIONS 6.8 - 6.19)

6.8. The mass spectrometer and evidence for nuclides

If atoms are ionised by heating or other they can be accelerated by a potential difference. If they are then entering a homogenous magnetic field there will be a magnetic force acting as a centripetal force on them when they move in a circular path. The radius of the circle (or here, half circle) will depend on their mass and velocity:

\[
a_08a \quad qvB = \frac{mv^2}{r} \Rightarrow m = \frac{qBr}{v}
\]

which gives \(qvB = \frac{mv^2}{r} \Rightarrow m = \frac{qBr}{v}\). If ions of the same element are used and the velocity of them when entering the magnetic field is the same then any difference in radius must be caused by differences in mass between nuclei of the same element. To arrange this we use a **mass spectrometer**:
1. **Ion source** (e.g. gas heated enough to be ionised) with accelerating voltage $V_{\text{acc}}$. Charges (ions) $q$ are accelerated by an electric field $E_{\text{acc}}$. In this acceleration they get a velocity given by $\frac{1}{2}mv^2 = qV$ where $V$ = the potential difference used.

2. **Speed filter**: This velocity can be controlled even more precisely by letting them into a chamber where we have a crossed **electric field $E$ and a magnetic field $B$**: here $E$ from up to down in the plane of the paper and $B$ out of the plane of the paper causing force which for charges $q$ with a suitable velocity $v$ cause opposite forces of equal magnitude:

- $F_{\text{electric}} = F_{\text{magnetic}}$ where $E = F_{\text{electric}}/q$ $\Rightarrow F_{\text{electric}} = qE$
- $qE = qvB$ which cancelling $q$ gives $v = E/B$

With collimators (two or more narrow slits or holes that the charges must pass through) only ions with precisely the right velocity $v$ can pass into the

3. **Separation chamber**: here the charges enter another magnetic field $B'$ but not electric field, so the magnetic force can act as a centripetal force giving

- $mv^2/r = qvB'$ and as before
- $m = qBr/v$ which with $v = E/B$ giving $m = qBB'r/E$

By letting the ions move in a half-circle with radius $r$ in the separation chamber they will hit a detection device (photographic film or other) in slightly different places if they have different masses. This can be used to find the precise mass of an ion and is used in organic chemistry to help find out the properties of unknown molecule; it has also been used to separate heavier $^{238}\text{U}$ - versions (isotopes) of uranium from lighter $^{235}\text{U}$ which can
be used for nuclear fuel or weapons. (More effective methods based on gas diffusion are more common today).

6.9. The nucleus

Atom number $Z$ and mass number $A$

With a mass spectrometer it can be shown that nuclei of the same chemical element can have different masses. Since this cannot be caused by different numbers of protons (which would result in a different number of electrons and therefore other chemical properties for the neutral atom) there must also be neutral particles in the nucleus. These are difficult to detect directly since most detection methods are based on the electric charge of a particle, but indirectly (detecting not the neutrons themselves but particles resulting from nuclear reactions caused by neutrons) neutrons were discovered in 1930.

- **nucleon** = proton or neutron
- **isotope** = versions of nuclei of the same element with the same number of protons but different numbers of neutrons

An atom of the element $X$ with the atom number $Z$ (= number of protons = number of electrons if neutral) can be symbolised:

$$^{A}{}_{Z}X$$

where $A$ = the mass number = the total number of protons ($Z$) and neutrons ($N$) so $A = Z + N$. Example:

$$^{238}{}_{92}U$$ (or before: U-238)

means a uranium atom with 92 protons (as all uranium atoms have) and a total of 238 nucleons, so the number of neutrons is $238-92 = 146$.

The unified mass unit

The mass of a nucleus is small and here often measured in the "atomic mass unit" where $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$ (by definition $1/12$ the mass of a C-atom). The masses of various particles and atoms (usually the mass not for the nucleus alone but for the neutral atom including $Z$ electrons) is found in various tables. Examples:

- electron $e$ \quad 0.000549 u
- neutron $n$ \quad 1.008665 u
- proton $p$ \quad 1.007276 u
- $^1H$ - atom \quad 1.007825 u
- $^4He$ - atom \quad 4.002602 u
- $^{12}C$ - atom \quad 12.00000 u (by definition)
- $^{16}O$ - atom \quad 15.994915 u

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Keeping the nucleus together: electric repulsion and strong interaction

In the nucleus we have positive charges which with the electric Coulomb force would repel each other and would make the nucleus split unless another force, the "nuclear force" or strong interaction kept it together. There are 4 basic forces or types of interaction:

- gravity
- electromagnetic
- strong interaction ("nuclear force")
- weak interaction (causes some types of radioactive decay)

The nuclear force is attractive at distances about the size of a nucleon and thereafter very quickly becomes weaker (so for larger nuclei one tends to need more and more nucleons to keep it stable; in He, C and O the stable isotopes have Z = N but in uranium Z = 92 and N = 146!). At distances shorter than the size of a nucleon it becomes repulsive, preventing the universe from collapsing.

6.10. Mass defect and binding energy

Mass defect and mass as a form of energy

Mass defect (missing mass): if we add the mass of the constituents and compare it to the mass of the whole neutral atom (including electrons), then some mass is missing.

Ex. for $^4_2$He the mass is 4.002602 u but if we add up

$$2m_e + 2m_p + 2m_n = 4.032982 \text{ u}$$

then some mass is missing; the difference is

$$4.032982 \text{ u} - 4.002602 \text{ u} = 0.03038 \text{ u}$$

According to relativity theory (later), mass and energy are two forms of the same thing. They follow:

$$E = mc^2$$

where c = the speed of light.

Mass can then also be measured in energy units, like the electronvolt. 1 u = 931.5 MeV (data booklet).

This missing mass is then a form of energy which may be released if the atom and especially the nucleus changes in such a way that more energy is missing. The missing energy can be called binding energy. It is usually calculated positive, although it could (maybe more reasonably) be given a negative sign, like the negative gravitational potential energy of an object near a planet. This can happen both in radioactive decay and nuclear fission and fusion (described later) all of which can produce heat energy.
6.11. Changing the nucleus I: natural radioactivity

In a nucleus, the repulsion between the positive protons is overcome by the nuclear force or strong interaction which is attractive at suitable distances, see above. This leads to a certain relation between the number of protons and neutrons being most stable, which can be described by a graph of the "stable valley":

![Graph of stable valley]

Nuclides outside the stable valley may approach it by radioactive decay, here primarily alpha and beta decay. The third type of decay, gamma decay, concerns a rearranging of the protons and neutrons in the nucleus without changing the number of them.

This decay means that a parent nucleus X is transformed into a daughter nucleus Y and one or more particles (a, b, ...) emitted. (For gamma decay, X = Y and only a massless photon is emitted).

\[ X \rightarrow Y + a + b + \ldots \]

The process can happen spontaneously in nature if energy is released, that is the mass of what we have after the decay is less than we had before. This energy is taken away as kinetic energy of the Y, a, b, .... The released energy or reaction energy or "Q-value" is for the decay \( X \rightarrow Y + a \)

\[ Q = (m_X - m_Y - m_a)c^2 \]

This is can happen spontaneously if the Q-value calculated as above is positive. If it is negative, a decay or other nuclear reaction may still be caused if a particle is shot at the target nucleus with a high enough kinetic energy to balance out a negative Q (see artificial transmutation below).
6.12. Types of radioactive decay

Alpha decay of nucleus X

\[ ^{A\ Z\ X} \rightarrow ^{A-4\ Z-2\ Y} + ^{4\ Z\ He} \]

The helium atom or in practice positive ion is called alpha particle, sometimes \(^4\ _2\alpha\).

Beta decay of nucleus X, type 1: \(\beta^-\) -decay

The most common type of beta decay is \(\beta^-\) -decay where electrons are emitted as a neutron in the X-nucleus turns into a proton and the emitted electron.

\[ ^{A\ Z\ X} \rightarrow ^{A\ Z+1\ Y} + ^{0\ -1e} + ^{0\ \nu\ \nu} \]

where the \(\nu\) indicates an antineutrino (more often symbolised with an overlined \(\nu\)), a (probably) massless particle whose existence was postulated since beta decay would violate the laws of momentum and energy conservation if X only split into Y and the electron. (What an antiparticle is, and why we think an antineutrino is emitted here instead of an ordinary neutrino we will return to in particle physics later).

That a third particle is emitted could in principle be discovered by noting that the Y and the electron do not leave in exactly opposite directions as they should if X was at rest before the decay but this is difficult to observe since a sample contains millions and millions of atoms decaying in random directions. It can however be observed since if only two particles were emitted, any one of them should have a kinetic energy which depends only on the masses of the two and the energy Q released (from the difference in mass between the particles before and after the decay). In reality the emitted electrons have many different ("a continuous spectrum of") kinetic energies.

Assume we call the Y object 2 and the emitted electron object one. Conservation of momentum (with X at rest) gives (primes for situation after, no primes for situation before decay):

\[ \text{a12a} \]
• \[ 0 = p_1 + p_2 = p'_1 + p'_2 \] so \[ p'_2 = - p'_1 \] and squaring \[ \Rightarrow p'^2_2 = p'^2_1 \]

Conservation of energy then gives that any kinetic energy after the collision equals Q or:
• \( E_{k1'} + E_{k2'} = Q \) which using \( E_k = \frac{p^2}{2m} \) can be written
• \( p'^2_1/2m_1 + p'^2_2/2m_2 = Q \) and then using \( p'^2_2 = p'^2_1 \)
• \( p'^2_1/2m_1 + p'^2_1/2m_2 = Q \), factorise out \( p'^2_1 \) so
• \( p'^2_1(1/2m_1 + 1/2m_2) = Q \)

Then make \( 2m_1m_2 \) the common denominator in the parenthesis and we get
• \( p'^2_1[(m_2 + m_1)/2m_1m_2] = Q \) so
• \( (p'^2_1/2m_1)[(m_2 + m_1)/m_2] = Q \) or
• \( E_{k1'}[(m_2 + m_1)/m_2] = Q \) and finally
• \( E_{k1'} = Qm_2/(m_2 + m_1) \)

so the kinetic energy of the electrons emitted should have a constant value for the beta decay of a given isotope - which they do not, experimentally. (The kinetic energy of the emitted particles is easier to measure, for example collimators and a magnetic field should if the kinetic energy was constant make them all move in a circle with a constant radius, like a mass spectrometer without the speed filter. They do not.)

**Beta decay of nucleus X, type 2 : \( \beta^+ \)-decay**

Another type of beta decay is one where a *positron*, the antiparticle of the electron (see particle physics later) is emitted; a particle with the same mass as the electron but the opposite electric charge. The third particle emitted here is an ordinary neutrino, not an antineutrino.

\[ ^{A}{Z}X \rightarrow ^{A}{Z-1}Y + ^0{+1}e + ^0{0}\nu \nu \]

**Beta decay of nucleus X, type 3 : Electron capture (EC)**

This is a case where the nucleus snatches an inner-shell electron (whose place is then filled by some electron higher up falling down, whose place is taken eventually by an electron from the environment. It may seem worrying to environmentalists that electrons can just be dumped into or stolen from the environment like that, but due to the enormous amount of electrons which exist in nature this is not a significant threat to biodiversity. It may comfort the reader to know that this file was generated using 100% recycled electrons). An ordinary neutrino, not an antineutrino, is also emitted.

\[ ^{A}{Z}X + ^0{+1}e \rightarrow ^{A}{Z-1}Y + ^0{0}\nu \]

**Gamma decay of X**

Here no particle other than a photon with very high energy \( E = hf \) is sent out. This can happen if the nucleus was excited, that is the nucleons not arranged as close to each other as possible, so they can release energy by "falling" closer to each other. That the nucleus
X has some of its parts in a higher energy state than the lowest possible - that it is "excited" - is denoted with the symbol X*.

\[ {}^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \text{photon} \]

The high energy and therefore high-frequency electron (or low-wavelength photon) has a higher frequency than X-rays.

**Other decays of X**

Besides alpha, beta and gamma decay there are other types of decay, most notably those where neutrons are emitted and those were "particles" larger than an alpha particle are emitted ("cluster radioactivity").

### 6.13. Nuclear energy levels

Inside a nucleus we have protons and and neutrons which are arranged in a structure not entirely known yet. It seems that some "chunks" of nucleons are more stable than others, namely those with "magic numbers" for Z or N = 2, 8, 20, 28, 50, 82 or 126. For example helium has Z = N = 2 (and A = 4) wherefore the helium nucleus or alpha particle is very stable. It may be possible to think of nuclei as consisting of these "chunks" arranged in some way, for example to think of some chunks orbiting others or other more or less stable parts of a nucleus. This structure is relevant only at a the nuclear level - for the outside world, e.g. the electrons in the atom or another atom the nucleus is one concentrated object.

We may then consider these "orbits" (or other features of the internal structure of the nucleus) to be quantized like the orbits of the electrons around the whole nucleus. They would have discrete energy levels, not continuous ones. Experimental support for this is:

- **alpha particles emitted** in decay only have certain discrete (kinetic) energies
- **gamma spectra are also discrete**, like the atomic spectra (of visible light, IR or UV) emitted when electrons fall between electron shells (maybe a chunk has fallen down to a lower orbit)

(Recall, however, that the beta particle energy spectra are not discrete but continuous!)
For this reason the (not yet complete) model of nuclear structure is called a "shell" model of the nucleus.

### 6.14. Effects and detection of radiation

**Ionising radiation**

Particles emitted in radioactive decay primarily affect matter by ionising the atoms of the target material. Alpha and beta particles are themselves electrically charged, gamma radiation can cause secondary beta radiation (electrons) by giving part of its energy to an electron and being scattered with a lower remaining energy and therefore frequency (a phenomenon called Compton scattering, not in the IB programme).

The same ionising of matter hit by radiation is used in detecting it; for gamma radiation other methods than those mentioned here are used (certain materials, such as tellurium-treated sodium iodide emit visible light after being hit by gamma and this faint light can be magnified with a photomultiplier).

**The G-M tube**

The Geiger-Müller tube consists of a cylinder-shaped cathode connected to the negative terminal of a rather high DC-voltage source, and inside it a positive anode wire. In the tube there is a gas (argon) at low pressure. When a charged particle passes it, some gas atoms are ionised and moved by the electric potential difference causing a weak current pulse which is amplified and fed to a loudspeaker and/or analogue indicator.

![Diagram of a Geiger-Müller tube](a14a)

**The ionization chamber**

This older and more primitive device in which the charged alpha or beta particles themselves are to be electrically attracted to the electrodes causing a current pulse.

**Cloud and bubble chambers**

Especially in particle physics, charged particles may also be detected by the track they leave in vapour about to condense into a liquid (compare to tracks in the air left by airplanes or Formula 1 racing cars) or the track of bubbles in a liquid about to start
boiling. Photographs of these tracks may show circular paths of cloud or bubble tracks if a magnetic field is applied; the radius of such a circle gives information about the energy of the particle.

### 6.15. Decay Calculations and Half-life

**Radioactive Decay Formulas**

Decay is a random process where we cannot say exactly when one atom splits, only the probability that it does in a certain time, for example one second. The decay can be described with

\[
\Delta N = -\lambda \Delta t
\]

where \( N \) = the number of suitable nuclei, \( \Delta t \) = the time period and \( \lambda \) = the decay probability (NOT any wavelength!). The minus shows that the number of nuclei of this type decreases as a result of the decay into something else.

It can be shown (mathematically; a differential equation where we have \( \frac{dN}{dt} = -\lambda N \), compare this to \( y'(x) = ky(x) \), with the solution \( y(x) = e^{kx} \)) that if we start with \( N_0 \) nuclei at the time \( t = 0 \) the number of nuclei left = \( N \) after a time \( t \) is

\[
N = N_0 e^{-\lambda t}
\]  

[DB p.8]

where \( e \) now means the Napier's number \( e = 2.718.... \)

For the activity = the number of decays per second = \( A = \frac{\Delta N}{\Delta t} \) we also have

\[
A = A_0 e^{-\lambda t}
\]  

[not in DB]

The unit of activity = 1 decay/second = 1s\(^{-1}\) = 1 becquerel = 1 Bq

Similar formulas can be shown to be true for the mass \( m \) of the radioactive atoms in a sample, or the number of moles \( n \):

\[
m = m_0 e^{-\lambda t} \quad \text{and} \quad n = n_0 e^{-\lambda t}
\]

**Half-life**

For the time it takes to go from \( N = N/2 \) we have \( N/2 = N e^{-\lambda t} \Rightarrow 1/2 = e^{-\lambda t} \)

\( \Rightarrow 2 = e^{\lambda t} \) which if we take the natural logarithm (the inverse function to \( e^{x} \)) of both sides gives \( \ln 2 = \ln e^{\lambda t} \Rightarrow \ln 2 = \lambda t \Rightarrow t = \ln 2 / \lambda \)

This time is called the **half-life**.
Measuring the half-life of a nuclide

With a detector like a GM-tube we can get information about the activity as a function of time for a radioactive sample. It should be an exponentially decreasing curve, and the half-life can be found by taking any point on the curve, reading its activity value, and moving to the right on the horizontal time axis until half the first activity value is found. The time moved is the half-life. (Note that the shape of this curve is independent of the initial number of nuclei, and the half-life of how many % of the decays the GM-tube registers).

Alternatively, we can plot ln A as a function of t:

- \[ A = A_0 e^{-\lambda t} \], taking natural logarithms gives
- \[ \ln A = \ln(A_0 e^{-\lambda t}) = \ln A_0 + \ln(e^{-\lambda t}) = \ln A_0 + (-\lambda t) \] so
- \[ \ln A = -\lambda t + \ln A_0 \], that is we get a straight line with the gradient = -\( \lambda \) and using the \( \lambda \) we find the half-life from \( T_{1/2} = \ln 2 / \lambda \).

The measurements are problematic if the unknown half-life is either very long (maybe billions of years) or very short.

**Very long half-lives:** We can then the \( \lambda \) (and consequently the \( T_{1/2} \)) from \( \Delta N = -\lambda N \Delta t \) if we know \( N \) (the number of radioactive atoms in the sample; this can be found via chemical calculations of how many moles of the radioactive substance we have) and \( \Delta N \), the number of atoms which have decayed in a certain measurement time \( \Delta t \). This is not the same as the number of ticks heard from the GM-tube (or other activity measurement apparatus) since it has a low "efficiency"; that is it gives a tick only for very so many.
hits. If this efficiency is found using other samples with a known activity, then the problem is solved.

**Very short half-lives:** If the nuclide decays very quickly it may have to be produced (maybe with artificial transmutation, see 6.16. below) in the detector rather transported to it. If the decaying atom (or particle) is formed in a particle accelerator, then the distance it travels before decaying indicates the "life" of an individual atom or particle; many such measurements give the average half-life.

**Decay chains**

If X decays into Y which also is radioactive, this may decay into Z with a new half-life until a stable (non-radioactive nucleus is reached).

**6.16. Changing the nucleus II: artificial transmutation**

In section 6.11 it was described that spontaneous natural decay can occur if the reaction energy or Q-value is positive, that is when we have more mass before than after and the decrease in mass turns into kinetic energy of the pieces left after the decay. A change of nucleus where the Q-value would be negative can be caused by accelerating a particle a to a high kinetic energy and letting it collide with the "target" X, for example as:

\[ X + a \rightarrow Y + b \]

This artificial (induced) transmutation can be done if

\[ (m_a + m_X)c^2 + E_{k,a} > (m_Y + m_b)c^2 \]

In this way it is possible to turn lead into gold and thus fulfil the old dream of the alchemists, but the price of gold produced in this way would exceed that of ordinary gold.

**6.17. Changing the nucleus III: fission and fusion**

**Binding energy per nucleon, fission and fusion**

If we calculate the lost mass = binding energy per nucleon for different atoms (in the example with He above we would have divided the result with 4 and multiplied with 931.5 to get it in MeV per nucleon) we can make a graph of binding energy per nucleon as a function of mass number.
From this graph we can suggest two possible ways to produce energy:

- **fusion** (merging together very small nuclei) or **fission** (splitting heavy nuclei) gives more missing mass per nucleon
- the total number of nucleons is the same
- this means we get more total missing mass
- this missing mass is converted to energy as \( E = mc^2 \) and released in the nuclear reaction

In principle, energy is released also in natural radioactive decay as kinetic energy (and consequently an increased temperature in) the products. This energy source is used in some space probes since it may work for several years without any moving parts that may need maintenance (the thermal energy is converted to electrical with a thermocouple which has a low efficiency but no moving parts).

Fission and fusion can supply much larger amounts of energy since

- the change in energy/nucleon is larger than in a decay (e.g. emitting an alpha particle would not move us as far to the left on the A-axis as a fission where the parent nucleus is split in parts of roughly half the size of the parent)
- they can be used in chain reactions, where particles emitted in one fission cause other fissions (for fusion the situation is somewhat different, see below).

**Fusion**

In nuclear fusion, two light nuclei are fused together producing a new one with less binding energy per nucleon than before, so energy is released. Since the nuclei to be fused are both positive and repel each other with the electrical Coulomb force they must be given an enormous kinetic energy to reach each other, wherefore fusion only occurs at
very high temperatures of millions of kelvins. Fusion reactions are the source of energy in the sun (gravitational contraction of a cloud of hydrogen produced the temperatures needed to start the fusion process) and are used in "H-bombs" or thermonuclear devices. In these, the high temperature needed is given by an igniting nuclear charge of the fission type. For peaceful production of energy attempts are being made to use either of these reactions:

- **DD-fusion (deuterium-deuterium):** $^2_1H + ^2_1H \rightarrow ^4_2He$
- **DT-fusion (deuterium-tritium)** $^2_1H + ^3_1H \rightarrow ^4_2He + ^1_0n$

DT-fusion is easier to use but has the drawback that supplies of tritium are limited (ca 1000 years?) and that the neutron radiation causes machinery parts to become radioactive (they only need to be stored for about a century for this to decay enough). DD-fusion is more difficult but D can be produced from seawater for millions of years. Note that unlike the fission reaction below the products cannot be reused to make a chain reaction; the energy (heat) produced may however be used indirectly to cause more reactions.

## 6.18. Fission chain reactions: bombs and power plants

### Nuclear fission chain reactions

Artificial transmutation is possible for many situations, but requires that a particle enters the nucleus without losing too much energy to electric repulsion by the nucleus (alpha) or the electrons (beta). Gamma radiation does not directly affect any A, Z or N values if absorbed. But neutrons can hit the nucleus more easily, and in some reactions they are also emitted, e.g.:

$$^{235}_{92}U + ^1_0n \rightarrow ^{90}_{38}Sr + ^{144}_{54}Xe + 2 ^1_0n$$

Several different reactions take place and only the average number of new neutrons is interesting, other possible reactions are e.g.

$$^{235}_{92}U + ^1_0n \rightarrow ^{92}_{36}Kr + ^{141}_{56}Ba + 3 ^1_0n \quad \text{and}$$

$$^{235}_{92}U + ^1_0n \rightarrow ^{88}_{36}Sr + ^{136}_{54}Xe + 12 ^1_0n$$

- The total mass and atomic numbers are conserved: e.g. $235 + 1 = 90 + 144 + 2 \times 1$ and $92 + 0 = 38 + 54 + 0$
- the reaction produces energy if the total mass decreases:

$$\text{mass decrease} = m_{\text{before}} - m_{\text{after}} = m_U + m_n - m_{Sr} - m_{Xe} - 2 m_n$$

Converted to energy, the mass decrease causes a released energy $Q$ as:

$$Q = (m_{\text{before}} - m_{\text{after}})c^2$$

and the reaction produces energy if $Q > 0$
• since the reaction produces two new neutrons, these can be used again to split new U-nuclei
• these can then split 4, 8, 16 and so on in an uncontrolled chain reaction (nuclear explosion)
• a controlled chain reaction can be obtained if the number of neutrons that on average can cause a new fission = the multiplication factor = 1

\[ \text{a18a} \]

• in a reactor, some neutrons are lost because they leak out of the reactor (in a bomb, to make sure enough of them can hit another U-nucleus there must be a big enough piece of U, the critical mass) but can be reflected back by suitable materials (may be other pieces of fuel and or the water coolant)

**Moderators and safety**

• the neutrons produced in the reaction have high speeds, but (for reasons not presented here) only slow neutrons can pass the $^{238}\text{U}$ - nuclei which are 97% of the uranium in a typical reactor (in natural uranium more than 99%)
• to make sure enough neutrons can pass through the $^{238}\text{U}$ and reach a $^{235}\text{U}$ which is suitable to be split, the neutrons must be slowed down by a moderator
• this is a material which is made of light nuclei such that when neutrons collide with it, they are slowed down (compare a billiard ball hitting another)
• if the moderator and the coolant (the water or other material which transports away the generated energy as heat) are different materials, the chain reaction can continue even if the cooling fails or the control rods (which absorb extra neutrons) fail, which means there is a risk for uncontrolled fission (meltdown)
• if the moderator and coolant are the same material (technically difficult), a loss of coolant means a loss of moderator and therefore the chain reaction stops. This "passive safety" is used in modern reactors.
• the other products of the reaction (Sr, Xe) absorb neutrons and when enough has formed in the uranium fuel, the chain reaction cannot continue and the fuel must be changed.

6.19. Particle physics

Real particles and virtual field (exchange) particles

Main types (things which exist and things which do not)

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<th>REAL PARTICLES</th>
<th>VIRTUAL PARTICLES</th>
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</thead>
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<td>QUARKS (as hadrons = either baryons or mesons)</td>
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</tbody>
</table>

The 'non-existing' field particles are playing a role similar to that of the electric or magnetic field lines: they do not exist, but describe how the existing real particles behave. They can also be compared to the virtual image points produced by a concave lens (see the Optics option). There is no source of light in the virtual image point, but the rays of light after passing the lens act as if there was one. This virtual image cannot be focused on a screen, and in a similar way the virtual field particles themselves cannot be detected. But even so, the distance from a virtual image to the lens can be calculated with the same equation as for real images. And for virtual field particles we can calculate their mass, charge and various quantum numbers just like for real particles - which in their interactions behave as if a virtual particle had been exchanged between them:
Forces and virtual exchange particles

<table>
<thead>
<tr>
<th>ELECTROMAGN.</th>
<th>STRONG/COLOR</th>
<th>WEAK</th>
<th>GRAVITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>photon</td>
<td>mesons/gluons</td>
<td>vector bosons W, Z</td>
<td>graviton?</td>
</tr>
</tbody>
</table>

Classification of real (observed) particles

<table>
<thead>
<tr>
<th></th>
<th>real</th>
<th>virtual</th>
</tr>
</thead>
<tbody>
<tr>
<td>leptons</td>
<td></td>
<td></td>
</tr>
<tr>
<td>quarks (as hadrons)</td>
<td></td>
<td>[otherwise same structure....]</td>
</tr>
<tr>
<td>baryons(qqq)</td>
<td>mesons(qq')</td>
<td></td>
</tr>
</tbody>
</table>

Leptons

<table>
<thead>
<tr>
<th>Gener. 1</th>
<th>Gener. 2</th>
<th>Gener. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>electron</td>
<td>el. neutrino</td>
</tr>
<tr>
<td>Symbol</td>
<td>e</td>
<td>ν_e</td>
</tr>
<tr>
<td>Charge</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>

Quarks

<table>
<thead>
<tr>
<th>Gener. 1</th>
<th>Gener. 2</th>
<th>Gener. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>up</td>
<td>down</td>
</tr>
<tr>
<td>Symbol</td>
<td>u</td>
<td>d</td>
</tr>
<tr>
<td>Charge</td>
<td>+2/3</td>
<td>-1/3</td>
</tr>
</tbody>
</table>

Quark aggregates = hadrons

Quarks exists in two forms of hadrons (a hadron is anything made up of quarks):

- **baryons** = 3 quarks or 3 three anti-quarks (see below for what an antiparticle is)
- **mesons** = 1 quark and 1 anti-quark

The common proton and neutron are baryons, aggregates of three quarks:

- **proton** = uud (two up-quarks and one down-quark); total charge $+2/3 + 2/3 - 1/3 = +3/3 = +1$ (times the elementary charge)
- **neutron** = udd (one up-quark and two down-quark) total charge $+2/3 - 1/3 - 1/3 = 0$
By quark **flavor** we mean the type of quarks: u,d,s,c, t or b.

The quarks also exist in different "versions" with properties assigned the term **"color"**; a quark can be "red", "green" or "blue". These terms have no relation to colours as wavelength or frequency intervals for photons of visible light. (Photons, with no rest mass, are neither quarks nor leptons; they are particles in some sense not enough to get a place in this scheme. Neutrinos on the other hand, though they may be massless are leptons (they may on the other hand have a rest mass that is very low)).

There seems to be a rule in the universe that free particles are always colourless or "white"; either as a baryon (red-green-blue) or as a meson which must then be an aggregate of quarks with opposite color value (red-antired). Isolated quarks have so far not been detected.

*The strong = color force*

The strong or **color force** is **one** force described at different levels - between a proton and a neutron it is the "strong" force keeping the atomic nucleus stable with mesons as exchange particles. But the proton, neutron and meson are themselves made of quarks which are kept together by the (same) "color" force, with gluons as virtual exchange particles. So the strong interaction between the proton and the neutron can alternatively be described as a case of color interaction with gluons (in and between the proton, neutron and meson) as exchange particles. There may also be particles made of only gluons ("glueballs"), but their existence is not yet confirmed.

(Compare this to the one and same electromagnetic force which can act between to ions but at the same time between the electrons and protons in the ion)

*The weak force*

This force plays a role in beta decay and is the only force neutrinos interact with (if they are massless).

*Antiparticles*

To every particle there is an antiparticle which has the same mass but opposite electric charge and other properties (various quantum numbers, such as lepton and baryon number; momentum). Antiparticles are "real" particles which can interact with other materia and be detected, not virtual particles (although there can be virtual antiparticles like there can be virtual ordinary particles).

- Antiparticles can be produced from a very high energy photon; to conserve total electric charge we always get both an ordinary particle and an antiparticle (**pair production**). The photon must have the energy
\[ E = hf = 2m_{\text{particle}}c^2 = 2m_{\text{antiparticle}}c^2 \]

- If an antiparticle meets an "ordinary" real particle, they are annihilated and their mass turns to energy as photons (sometimes also other particles). Antiparticles can therefore not be stored in any ordinary container - they would be annihilated at contact with a wall atom - but can be temporarily contained with electromagnetic fields. Since large amounts of energy are released in an annihilation they are used in high-energy particle physics where various otherwise unusual (and often very short-lived) particles are produced.

**Beta decay revised**

Certain quantum numbers, such as the number of baryons or leptons seem to be conserved in all reactions. An example of this is the lepton number \( L \), defined so that:

\[ L = 1 \text{ for leptons, } L = -1 \text{ for antileptons, } L = 0 \text{ for others} \]

Let us now review the \( \beta^- \) decay:

\[ ^{A}_{Z} X \rightarrow ^{A}_{Z+1} Y + ^{0}_{-1} e + ^{0}_{0} \nu \]

Before the reaction we have no leptons (in the decaying nucleus \( X \); there are electrons in the atomic electron shell but they are not part of the decay proper). After the decay we have the daughter nucleus \( Y \) (same story) and the electron (a lepton) and the antineutrino (an antilepton). The total lepton number is therefore 0 both before and after.

**Unified theories**

It can be shown that the electromagnetic and weak forces are two forms of one force, and further that this "electroweak force" and the strong force all are the same force in "grand unified theories" (GUT). Attempts have been made to describe all four fundamental forces (including gravity) as one force (TOE = theory of everything) but so far the situation is not entirely clear about this.

### 6.20. Accelerators

Much of the experimental particle physics is done with accelerators, where particles are given enormous kinetic energies and in collisions may produce new particles. To accelerate a particle it must be electrically charged (atoms can be accelerated as ions). Electric fields are used to increase the speed and electric or magnetic fields to keep the accelerated particle in the desired path. The kinetic energy received by a particle accelerated through the potential difference (voltage) \( V \) is:

\[ E_k = \frac{1}{2}mv^2 = qV \]

To reach very high kinetic energies high voltages are used, and the voltage is used several times. Two types of accelerators are:
• **linear accelerators**, where an alternating voltage is connected to a series of tubes so that the particle is always repelled by the previous tube and attracted by the following. The voltage changes polarity while the particle passes through the tube. Since the particle will travel faster and faster and pass a given length of tube in a shorter time, they have to become longer and longer.

![Diagram of linear accelerator](image1)

• **cyclotrons**, which are based on keeping the particles in circular paths with a magnetic field providing centripetal force. The circular motion takes place in D-shaped half-circles with a small gap between; over this gap a high AC voltage is applied so that they are accelerated every time they pass the gap. The radius of the path increases, but the "**cyclotron frequency**" remains the same:

$$F_{c} = F_{m} = \frac{mv^{2}}{r} = qvB = \frac{mv}{r} = qB \Rightarrow v = \frac{rqB}{m}$$

And since $$v = \frac{2\pi r}{T} = 2\pi f$$ we get:

$$f_{cyclotron} = \frac{qB}{2\pi m}$$

For very high speeds, the cyclotron frequency needs to be adjusted during the acceleration because of relativistic effects (this is done in a **synchrocyclotron**).